THE ASSESSMENT OF MODEL ADEQUACY

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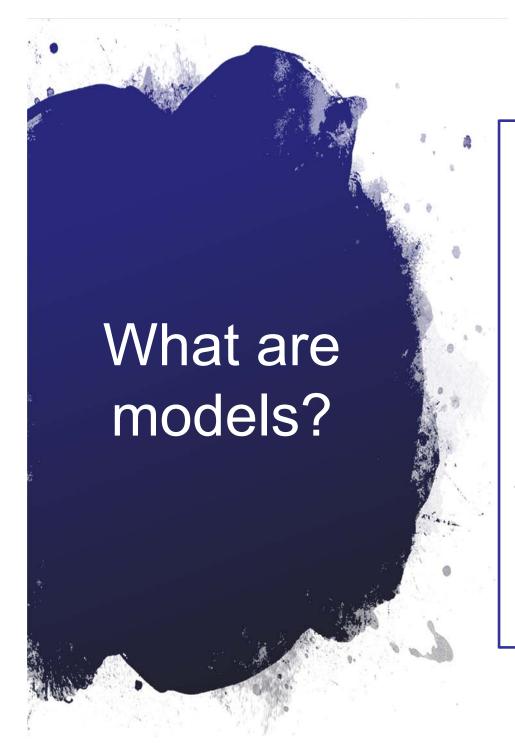
Review

Assessment of the adequacy of mathematical models

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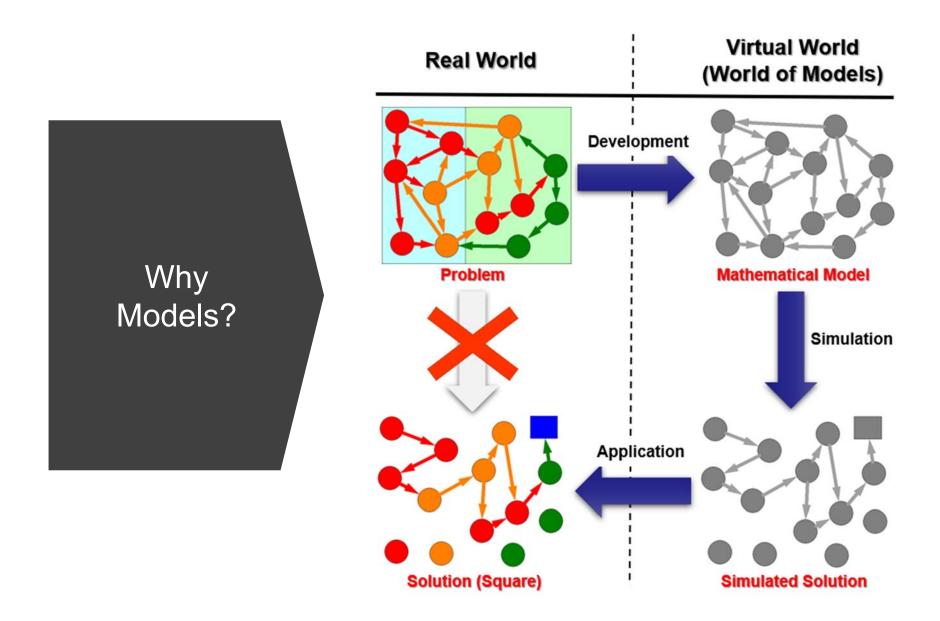
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http://www.nutritionmodels.com/mes.html



"Mathematical models (MM) are mental conceptualizations, enclosed in a virtual domain, whose purpose is to translate real-life situations into mathematical formulations (symbolically or numerically) to describe existing patterns or forecast future behaviors in the real-life situations"

-- Tedeschi (2019, 97:1921 J. Anim. Sci.)



"All models are wrong (false), but some are useful"

-- Box (1979)

- Understand and acceptance:
 - To strengthen the modeling process
 - To be more resilient to pitfalls during development and evaluation

- Improvement of the current model
- Understand the complex behavior of a phenomena by identifying small patterns in the process

"In systems thinking, the understanding that models are wrong and humility about the limitations of our knowledge is essential in creating an environment [model] in which we can learn about the complexity of systems in which we are embedded"

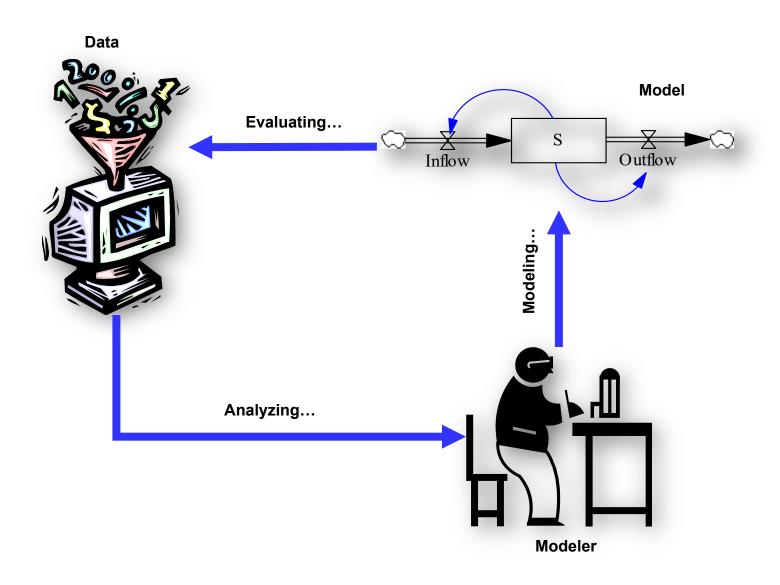


John Sterman

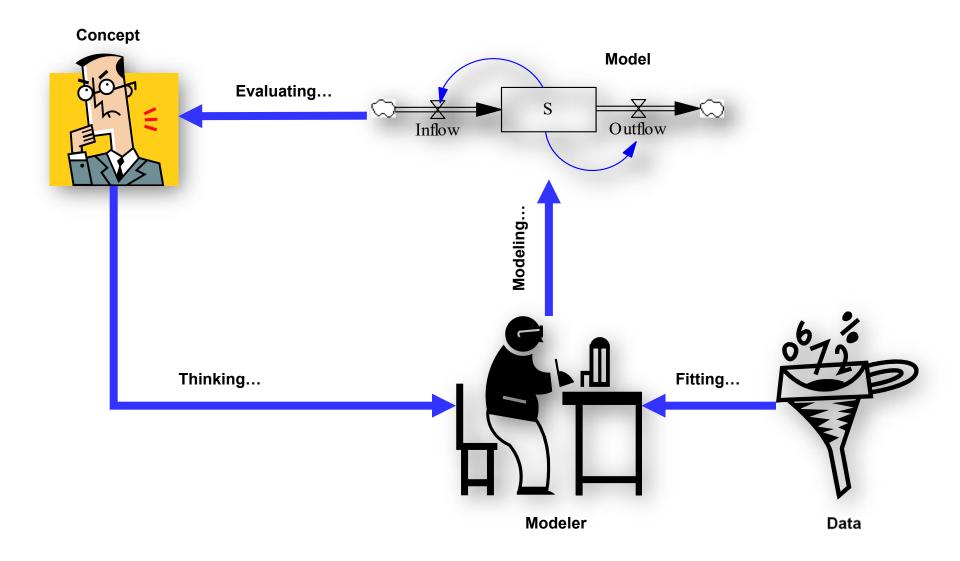
Processes for Model Development using Systems Thinking

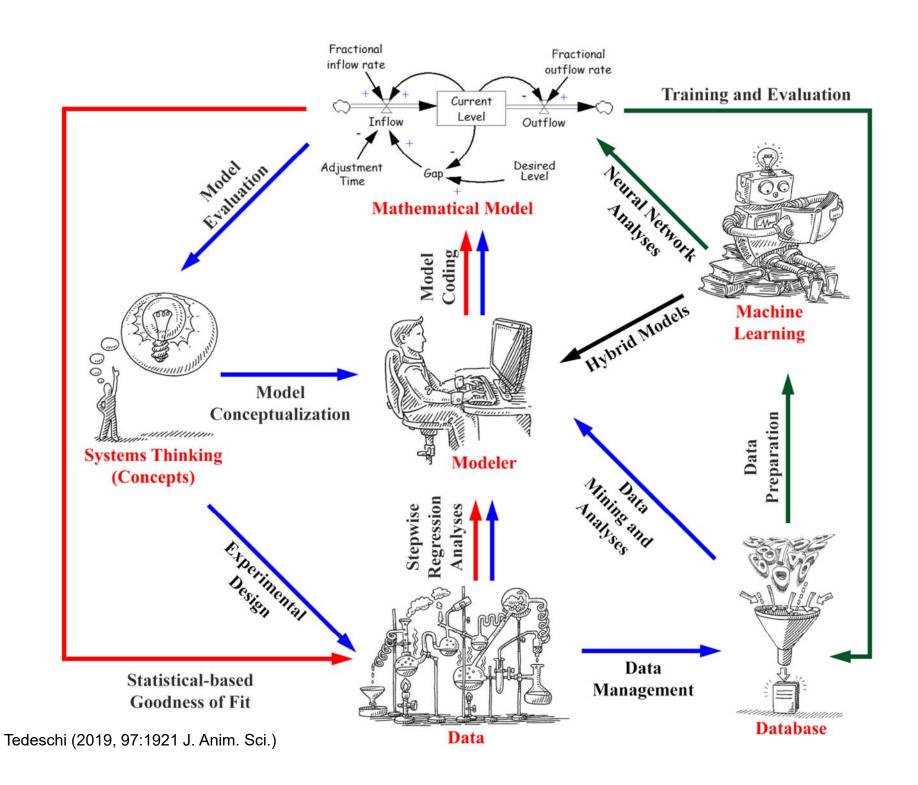


Empirical or Relational Models



Conceptual or Theoretical Models





Model Evaluation



"Model testing is often designed to demonstrate the rightness of a model and the tests are typically presented as evidences to promote its acceptance and usability"



John Sterman

-- Sterman (2002)



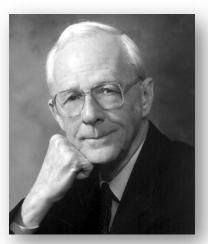
- Models cannot be validated
 - It is impossible to prove that all components of models or real systems are true or correct
- Models can never mimic the reality since they are representation of it
 - Some models can be programmed to predict quantities that cannot be measured in real systems

Models can be evaluated!

- Models can be <u>evaluated or tested</u>, but never validated
 - Validation means "having a conclusion correctly derived from premises"
 - Verification means "establishment of the truth, accuracy, or reality of"
- Calibration means model fine tuning or fitting; it is the estimation of values or parameters or unmeasured variables

"Validity of a mathematical model has to be judged by its sustainability for a particular purpose; that means, it is a valid and sound model if it accomplishes what is expected of it"

-- Forrester (1961)



Jay Forrester

DEFINITIONS OF EVALUATION

Shaeffer (1980)

- Model examination
- Algorithm examination
- Data evaluation
- Sensitivity analysis
- Validation studies
- Code comparison studies

Hamilton (1991)

- Verification
 - Design, programming, and checking processes of the program
- Sensitivity Analysis
 - Behavior of each component of the model
- Evaluation
 - Comparison of model outcomes with real data

Evaluating Errors



TWO-WAY DECISION PROCESS

	Model Predictions	
Decision	Correct	Wrong
Reject	Type I Error (α)	Correct (1 - β)
Accept	Correct (1 - α)	Type II Error (β)

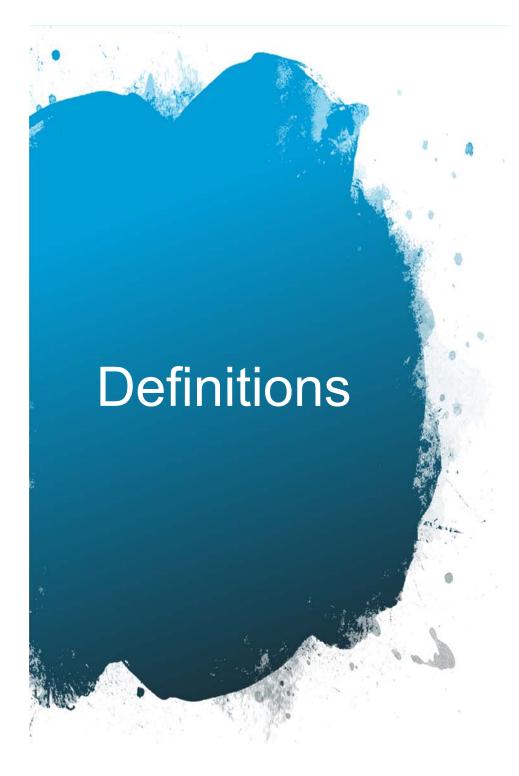


How does it happen?

- Type I Error (α): Rejecting an appropriate model
 - Biased or incorrect observations are chosen to evaluate a model
- Type II Error (β): Accepting a wrong model
 - Biased or incorrect observations are used to develop and evaluate a model
 - Conceptual model cannot be tested because lack of data

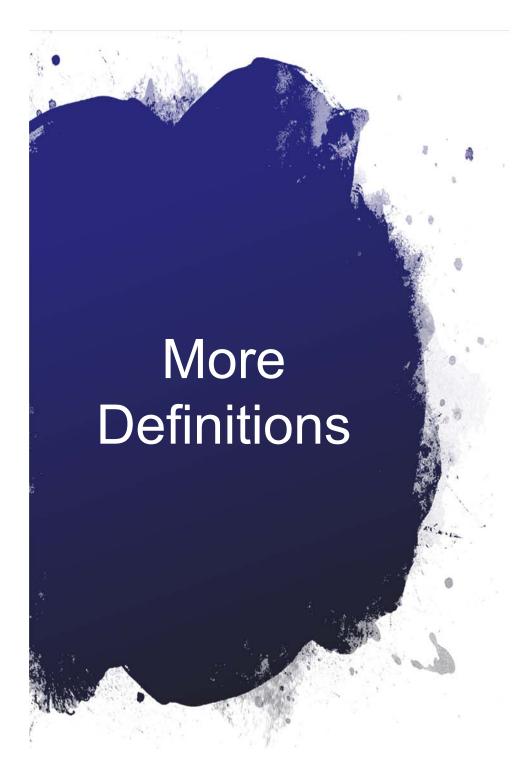
Accuracy x Precision





Accuracy

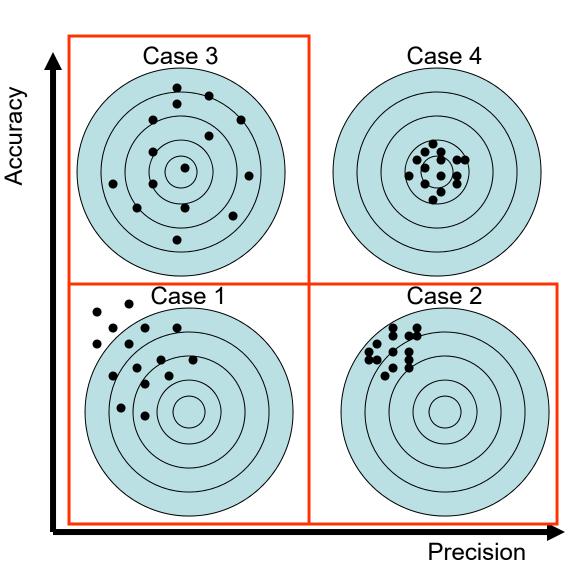
- It measures how closely model- predicted values are to the true values
- Ability to predict the right values
- Precision
 - It measures how closely individual model-predicted values are within each other
 - Ability to predict similar values consistently



- Inaccuracy or bias
 - Systematic deviation from the truth
- Imprecision or uncertainty
 - Magnitude of the scatter about the average mean

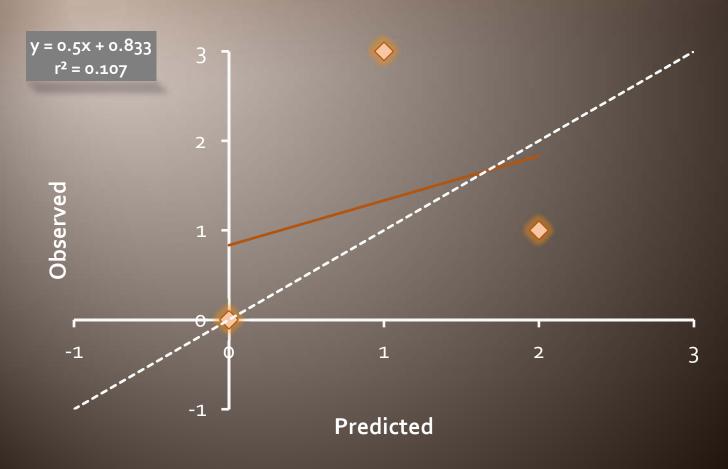
Imprecise or Uncertain

Inaccurate or biased

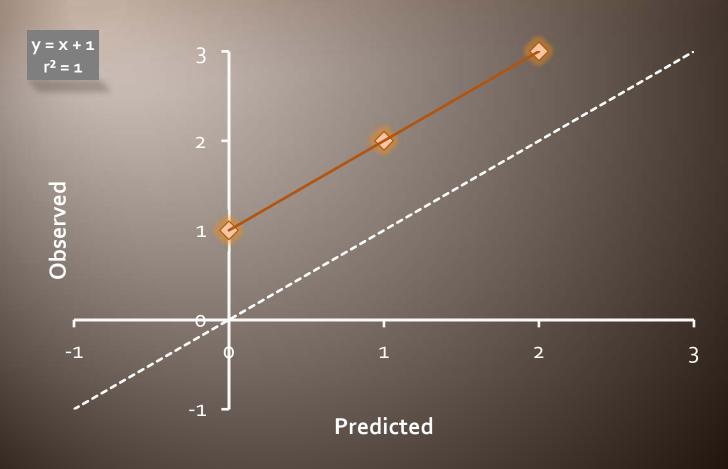


Tedeschi (2006: 89:225 Ag. Syst.)

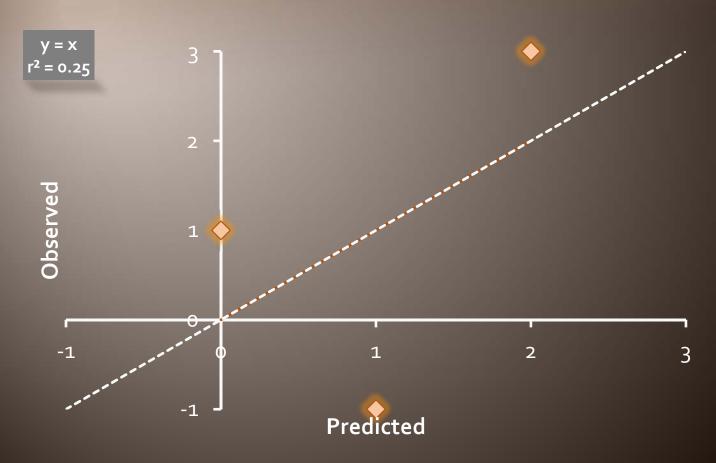
Case 1 - Precision V Accuracy



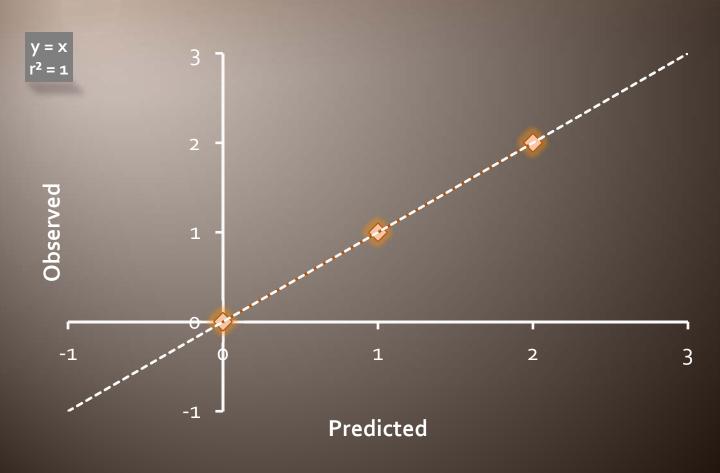
Case 2 - Precision \ Accuracy



Case 3 - Precision 1 Accuracy



Case 4 - 1 Precision 1 Accuracy





- Accuracy and Precision are independent
 - ↑ Accuracy does not imply ↑ Precision and vice-versa
- Imprecise model can get the right value using large number of data points (e.g. case 3)
- True mean is irrelevant for model comparison if the model is consistent (e.g. case 2)

Techniques for Model Evaluation: Regression Analysis



¿ Y-axis or X-axis? We regress the observed data (Y-axis) on the model-predicted (X-axis)

When using least-squares technique the vertical difference is minimized to estimate the parameters

Observed data has the random error, not the model-predicted values assuming deterministic model

Even stochastic models can be rerun several times, decreasing the error

Why linear regression?

- Hypothesis is that when regression Y (Obs) on f(X_{1,...} X_p)_i (Model-Pred), a perfect prediction would have intercept = 0 and slope = 1
- Little interest since the predicted value (by the linear regression) is useless in evaluating the mathematical model
- r² is irrelevant since one does not intend to make predictions using the fitted line!
 - May use it to adjust for model imprecision!

Assumptions for LR

The X-axis values are known without errors (deterministic)

The Y-axis values MUST be independent, random, and homoscedastic

Residuals are independent and identically distributed $\sim N(0,\sigma^2)$

Caution about r²

A high coefficient of correlation (r) does not indicate that useful predictions can be made by a given mathematical model since it measures precision not accuracy

A high r does not imply the estimated line is a good fit (curvilinear)

An r near zero does not indicate that observed and model-predicted are not correlated since they may have a curvilinear shape

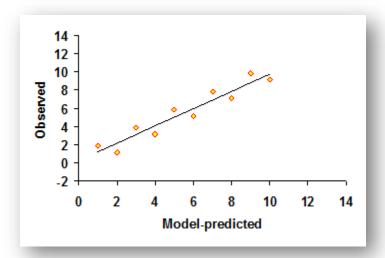
MEAN SQUARE ERROR (MSE)

- Also known as residual mean square or standard error of the estimate
- This statistic may be used to compare model 'validity' when comparing models

$$MSE = \frac{\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2}{n-2}$$

$$MSE = \frac{s_Y^2 \times (n-1) \times (1-r^2)}{n-2}$$

Comparison of Model Prediction



•
$$Y_1 = a + b \times X \pm N(0,1)_{\alpha=0.2}$$

•
$$a = 0.28 \pm 0.63$$

•
$$b = 0.95 \pm 0.10$$

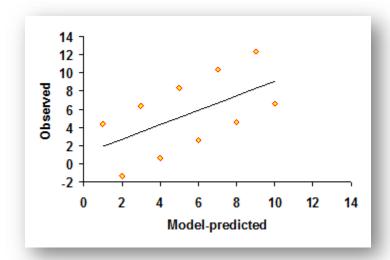
•
$$P(a=0) = 0.67$$

•
$$P(b=1) = 0.63$$

•
$$P(a=0 \& b=1) = 0.90$$

•
$$r^2 = 0.92$$

•
$$MSE = 0.89$$



•
$$Y_2 = a + b \times X \pm N(0,4)_{\alpha=0,2}$$

•
$$a = 1.12 \pm 2.53$$

•
$$b = 0.80 \pm 0.41$$

•
$$P(a=0) = 0.67$$

•
$$P(b=1) = 0.63$$

•
$$P(a=0 \& b=1) = 0.90$$

•
$$r^2 = 0.32$$

Concerns about LR

Assumptions of normality and homoscedasticity are rarely satisfied

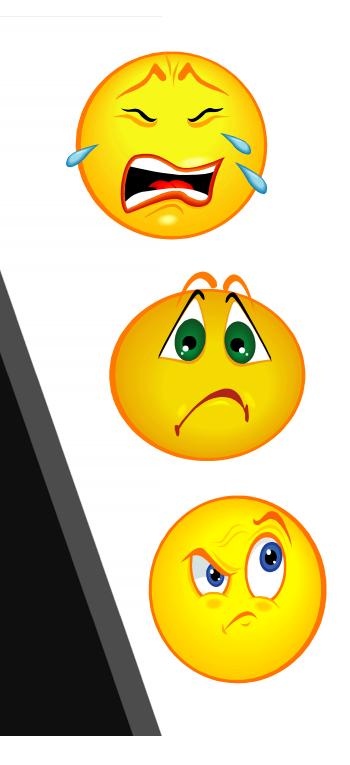
Ambiguous results depending on the scatter of the data

Regression lacks sensitivity to distinguish between random clouds and data points

Stochastic models require different technique to derive the parameters

Is r² a good indicator of adequacy?

- r² measures how far (close) the observations (Y values) deviate from the best-fit regression
- The best-fit regression IS NOT the model-predicted values
- r² does not distinguish between:
 - observed and modelpredicted values strongly agree
 - a strong linear relationship exists but the measurements do not agree



Tedeschi (2006: 89:225 Ag. Syst.)

Fitting Errors: Analysis of Deviation



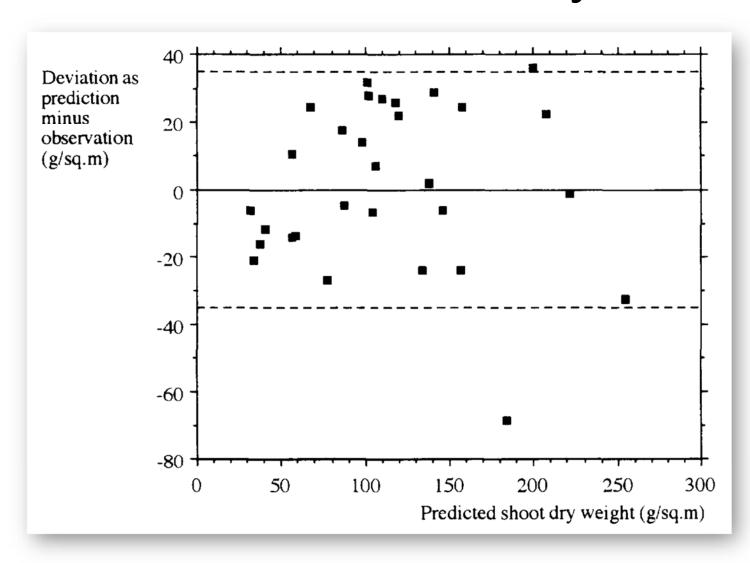
Analysis of Oeviation

Empirical but powerful analysis

Deviation is the difference between **model-predicted** minus **observed** values

Usually, an acceptable range is used to accept or not the model performance

Deviation Plot Analysis

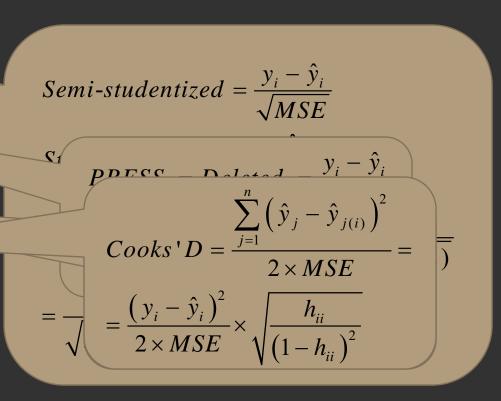


Fitting Errors: <u>Extreme and Influential Points</u>



FITTING ERRORS

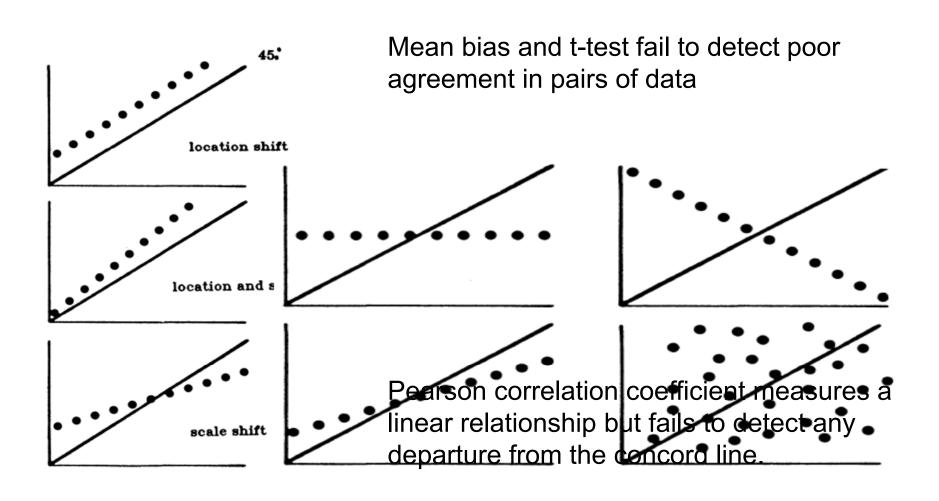
- Extreme Points
 - Leverage
 - Studentized residue
 - PRESS_p
- Influential Points
 - DFFITS
 - Cook's distance



CONCORDANCE CORRELATION COEFFICIENT (CCC)



Failure of Agreement Measures



¿What is CCC?

CCC aka reproducibility index

Are the model-predicted values precise and accurate at the same time across a range and are tightly amalgamated along the unity line through the origin?

CCC accounts for precision and accuracy at the same time

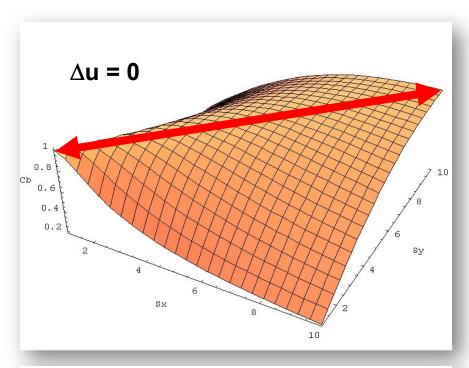
Proposed initially by Krippendorff (1970) and modified by Lin (1989)

HOW IS CCC COMPUTED?

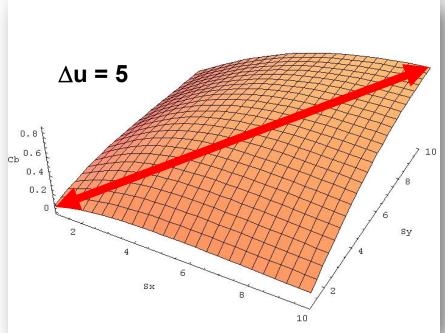
$$\hat{\rho}_{c} = \frac{2 \times s_{f(X_{1},...,X_{p})Y}}{s_{Y}^{2} + s_{f(X_{1},...,X_{p})}^{2} + (\overline{Y} - \overline{f}(X_{1},...,X_{p}))^{2}}$$

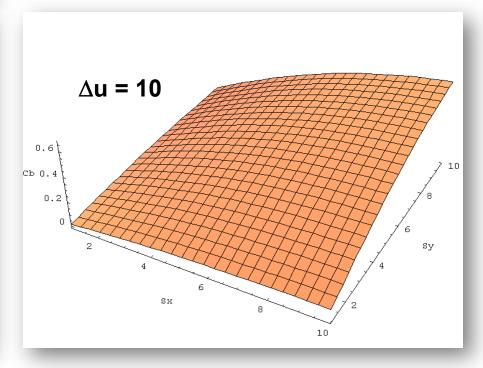
$$\hat{\rho}_{c} = \hat{\rho} \times C_{b} \quad \text{and} \quad C_{b} = \frac{2}{\left[v + \frac{1}{v} + \mu^{2}\right]}$$

$$v = \begin{cases} = \frac{\sigma_{1}}{\sigma_{2}} \text{ for population} \\ = \frac{s_{Y}}{s_{f(X_{1},...,X_{p})}} \text{ for sample} \end{cases} \quad \text{and} \quad \mu = \begin{cases} = \frac{\mu_{1} - \mu_{2}}{\sqrt{\sigma_{1}\sigma_{2}}} \text{ for population} \\ = \frac{\overline{Y} - \overline{f}(X_{1},...,X_{p})}{\sqrt{s_{Y}s_{f(X_{1},...,X_{p})}}} \text{ for sample} \end{cases}$$



Effects of Δu and Δv on Accuracy (Cb)



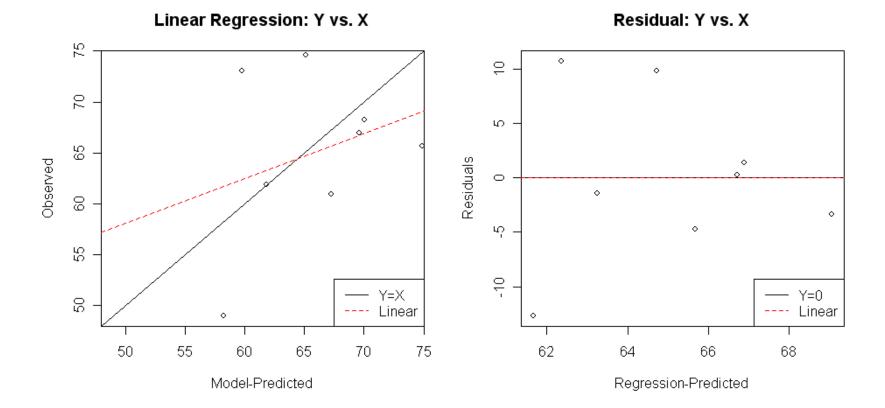


Example – Moment Statistics

```
• X Abs Mean :
             4.59500
                          Y Abs Mean :
                                         5.83031
• X Min
               58.24000
                          Y Min
                                    : 49.00000
 X Max
               74.90000
                          Y Max
                                       74.56000
               65.84750
                                    : 65.04375
 X Mean
                          Y Mean
 X Median :
               66.20000
                          Y Median
                                        66.32000
                          Y Variance : 64.80948
• X Variance : 32.34248
 X Std. Dev.:
             5.68704
                          Y Std. Dev.: 8.05043
• X Skewness:
             0.09757
                          Y Skewness: -0.67381
• X Kurtosis : 1.45303
                          Y Kurtosis : 2.35381
• X - Y Mean : 0.80375
                          X - Y Var : 68.47100
• Covariance : 12.54792
```

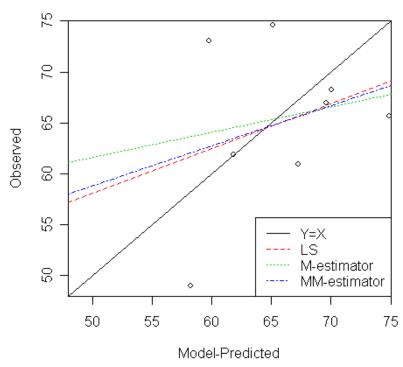
• Cb = 0.94

Example



Example – Robust regression

LS and Robust Regression: Y vs. X



Limitations of CCC

Assumes that each pair of data point are interchangeable, that means, the order of the data point does not matter; there is no covariance

Nickerson (1997) suggested an adaptation to the CCC

An improved CCC estimate

- CCC uses squared perpendicular distance (Y₁ Y₂)² of any paired data point to the unity line
- Unfortunately, it measures only how close the data point is to the unity line and not which direction it goes

An improved CCC estimate

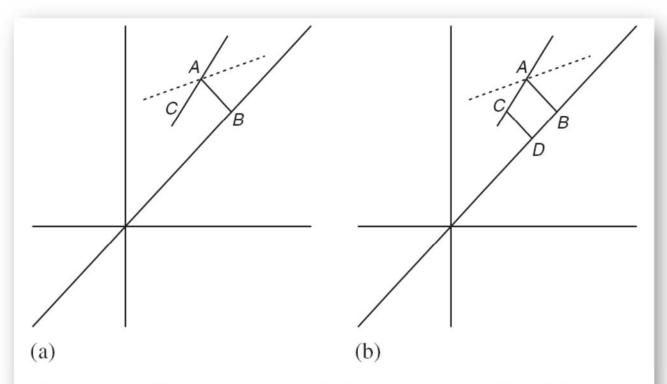


Figure 1. Comparison of the two criteria: (a) Lin's criteria; (b) new criterion.

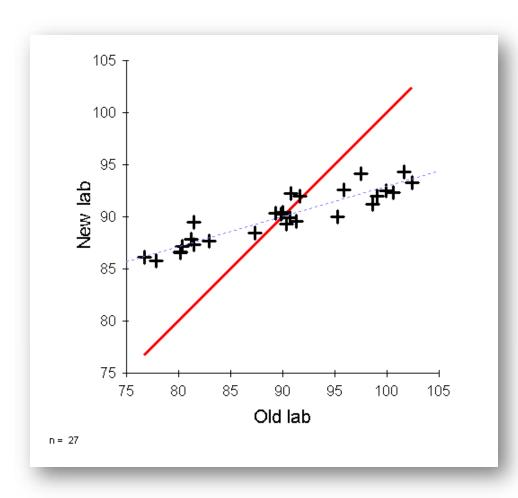
AN IMPROVED CCC ESTIMATE

- It is a quadratic area function of ρ whereas in Lin's it is quadratic distance function of ρ
- Accuracy (A_{ρ}) includes ρ whereas in Lin's (C_b) it does not

$$A_{\rho} = \frac{4 \times \left(\frac{s_{f(X_{1},\dots,X_{p})}}{s_{Y}}\right) - \hat{\rho} \times \left[1 + \left(\frac{s_{f(X_{1},\dots,X_{p})}}{s_{Y}}\right)^{2}\right]}{\left(2 - \hat{\rho}\right) \times \left[1 + \left(\frac{s_{f(X_{1},\dots,X_{p})}}{s_{Y}}\right)^{2}\right] \times \left(\frac{\overline{Y} - \overline{f}(X_{1},\dots,X_{p})}{s_{Y}}\right)^{2}}$$

$$\hat{\gamma}_{\rho} = \hat{\rho} \times A_{\rho}$$

Comparison Lin's x Liao's CCC



- Lin's CCC
 - $-C_{b} = 0.571$
 - $r_c = 0.527$
- Liao's CCC
 - $-A_r(C_b) = 0.205$
 - $-G_r(r_c) = 0.189$
- Chinchilli's CCC
 - $-GCCC_{w} = 0.179$





MEAN BIAS (MB)

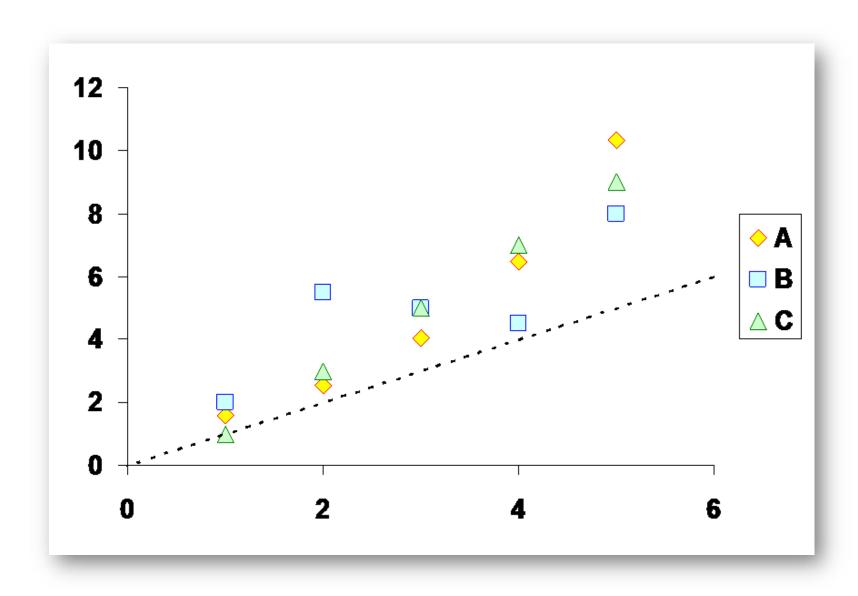
- Likely to be the oldest and most used statistic to assess model accuracy
- Likely to be the weakest one too!

$$MB = \frac{\sum_{i=1}^{n} (Y_i - f(X_1, ..., X_p)_i)}{n}$$

$$t_{MB} = \frac{MB}{\sqrt{\sum_{i=1}^{n} ((Y_i - f(X_1, ..., X_p)_i) - MB)^2}}$$

$$\sqrt{\frac{\sum_{i=1}^{n} ((Y_i - f(X_1, ..., X_p)_i) - MB)^2}{n \times (n-1)}}$$

Which model has the lowest MB?



- All models (A, B, and C) have the same MB = 2
- t-test for Model A (exponential)
 - Assuming $\sigma_1 = \sigma_2$: P = 0.29
 - Assuming $\sigma_1 \neq \sigma_2$: P = 0.28
 - Assuming covariance: P = 0.09
- t-test for Model B
 - Assuming $\sigma_1 = \sigma_2$: P = 0.14
 - Assuming $\sigma_1 \neq \sigma_2$: P = 0.13
 - Assuming covariance: P = 0.02
- t-test for Model C (linear)
 - Assuming $\sigma_1 = \sigma_2$: P = 0.25
 - Assuming $\sigma_1 \neq \sigma_2$: P = 0.24
 - Assuming covariance: P = 0.05



Mean bias

- Must be adjusted for covariance!
- Rejection rates of the H₀
 hypothesis increases as
 correlated errors increase
- Cannot be used as the main statistics for model evaluation

RESISTANT R²

- Resistant means it is insensible to outliers or extreme points
- Uses the median instead of mean

$$r_r^2 = 1 - \left(\frac{\mathbf{M}\left(\left|Y_i - \hat{Y}_i\right|\right)}{\mathbf{M}\left(\left|Y_i - \overline{Y}\right|\right)}\right)^2$$

$$\sum_{i=1}^{n} \left(\left|Y_i - \overline{Y}\right|\right)$$

MODELING EFFICIENCY (MEF)

• Proportion of variation explained by the line Y = $f(X_1,...,X_p)$; range $[-\infty \text{ to 1}]$; MEF = 1 is better

$$MEF = \frac{\left(\sum_{i=1}^{n} (Y_{i} - \overline{Y})^{2} - \sum_{i=1}^{n} (Y_{i} - f(X_{1}, ..., X_{p})_{i})^{2}\right)}{\sum_{i=1}^{n} (Y_{i} - \overline{Y})^{2}} = 1 - \frac{\sum_{i=1}^{n} (Y_{i} - f(X_{1}, ..., X_{p})_{i})^{2}}{\sum_{i=1}^{n} (Y_{i} - \overline{Y})^{2}}$$

$$r = 1 - \frac{\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2}{\sum_{i=1}^{n} (Y_i - \overline{Y})^2} = \frac{S_{f(X_1, \dots, X_p)Y}}{S_Y \times S_{f(X_1, \dots, X_p)}}$$

NASH-SUTCLIFFE MODEL EFFICIENCY COEFFICIENT (= MEF)

- Nash–Sutcliffe efficiency can range from -∞ to 1
 - NSE = 1 → corresponds to a perfect match of modeled values to the observed data
 - NSE = o → indicates that the model predictions are as accurate as the mean of the observed data
 - NSE < 0 → observed mean is a better predictor than the model or, in other words, when the residual variance (described by the numerator in the expression above), is larger than the data variance (described by the denominator).

$$NSE = 1 - rac{\sum_{t=1}^{T} \left(Q_m^t - Q_o^t
ight)^2}{\sum_{t=1}^{T} \left(Q_o^t - \overline{Q_o}
ight)^2}$$

COEFFICIENT OF DETERMINATION (CD)

- Ratio of total variance of observed data to the squared of the difference between model-predicted and mean of observed
- It is the proportion of the total variance of the observed values explained by the predicted data
- CD = 1 is better

$$CD = \frac{\sum_{i=1}^{n} (Y_i - \overline{Y})^2}{\sum_{i=1}^{n} (f(X_1, ..., X_p)_i - \overline{Y})^2}$$

Mean Square Error of Prediction (MSEP)



MSE x MSEP

MSE assesses the precision of the fitted linear regression using the difference between observed and regression-predicted values

MSEP consists the difference between observed and model-predicted values

MSEP X MSE

$$MSE = \frac{\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2}{n-2}$$

$$MSEP = \frac{\sum_{i=1}^{n} (Y_i - f(X_1, ..., X_p)_i)^2}{n}$$



Limitations of MSEP

- Removes the negative sign
- Weights the deviation by their squares, thus giving more influence to larger data points
- Does not provide information about model precision

DECOMPOSITION OF MSEP

 Expanded MSEP equation and solved for known linear measures of linear regression (Theil, 1961)

$$MSEP = \frac{\sum_{i=1}^{n} \left(Y_{i} - f(X_{1}, ..., X_{p})_{i}\right)^{2}}{n}$$

$$MSEP = \frac{\sum_{i=1}^{n} \left[\left(\overline{f}(X_{1}, ..., X_{p}) - \overline{Y}\right) + \left(f(X_{1}, ..., X_{p})_{i} - \overline{f}(X_{1}, ..., X_{p})\right) - \left(Y_{i} - \overline{Y}\right)\right]^{2}}{n}$$

$$MSEP = \left(\overline{f}(X_{1}, ..., X_{p}) - \overline{Y}\right)^{2} + s_{f(X_{1}, ..., X_{p})}^{2} + s_{Y}^{2} - 2 \cdot r \cdot s_{f(X_{1}, ..., X_{p})} \cdot s_{Y}$$

Understanding MSEP

$$\begin{split} \textit{MSEP}_1 &= \left(\overline{f}(X_1, ..., X_p) - \overline{Y}\right)^2 + \left(s_{f(X_1, ..., X_p)} - s_Y\right)^2 + 2 \cdot (1 - r) \cdot s_{f(X_1, ..., X_p)} \cdot s_Y \\ \textit{MSEP}_2 &= \left(\overline{f}(X_1, ..., X_p) - \overline{Y}\right)^2 + \left(s_{f(X_1, ..., X_p)} - r \cdot s_Y\right)^2 + (1 - r^2) \cdot s_Y^2 \\ \textit{MSEP}_3 &= \left(\overline{f}(X_1, ..., X_p) - \overline{Y}\right)^2 + s_{f(X_1, ..., X_p)}^2 \cdot (1 - b)^2 + (1 - r^2) \cdot s_Y^2 \end{split}$$

Inequality Proportions	Equations	Descriptions
U^{M}	$\left(\overline{f}(X_1,,X_p) - \overline{Y}\right)^2 / MSEP$	Mean bias
U ^s	$\left(s_{f(X_{1},\ldots,X_{p})}-s_{Y}\right)^{2}/MSEP$	Unequal variances
U^{C}	$2 \times (1-r) \times s_{f(X_1,\dots,X_p)} \times s_Y / MSEP$	Incomplete (co)variation
U^R	$s_{f(X_1,\ldots,X_p)}^2 \times (1-b)^2 / MSEP$	Systematic or slope bias
U^{D}	$(1-r^2)\times s_Y^2/MSEP$	Random errors
^a Note that $U^{M} + U^{S} + U^{C} = U^{M} + U^{R} + U^{D} = 1$		

Understanding MSEP

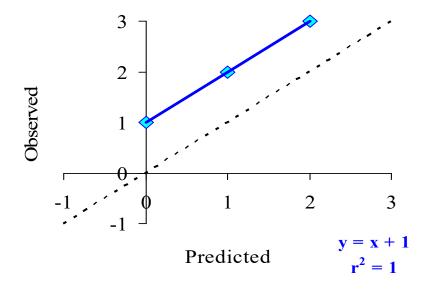
Mean bias indicate the error in central tendency

Systematic bias indicate how much the regression deviates from Y = X line, that means, errors due to regression

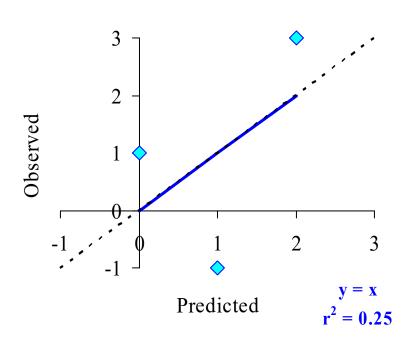
Random errors indicate the unexplained variation that cannot be accounted for by the relationship

Case 2 - ↑ Precision ↓ Accuracy

- MSEP = 1
- Mean Bias = 100%



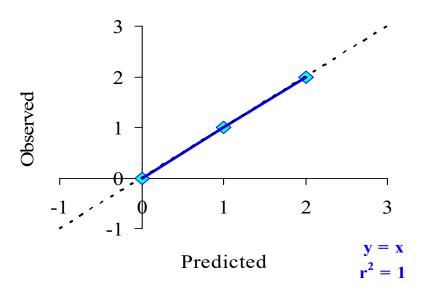
Case 3 - ↓ Precision ↑ Accuracy



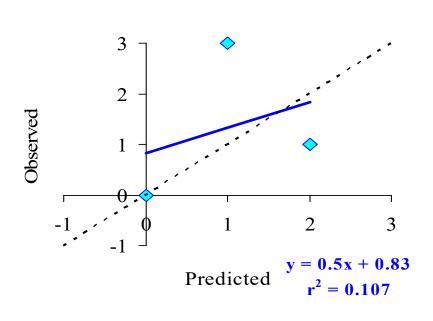
- MSEP = 2
- Mean bias = 0
- Slope bias = 0
- Random = 100%
- Unequal variance = 33.3%
- Incomplete (co)variation = 66.7%

Case 4 - ↑ Precision ↑ Accuracy

• MSEP = 0



Case 1 - ↓ Precision ↓ Accuracy



- MSEP = 1.67
- Mean bias = 6.7%
- Slope bias = 10%
- Random = 83.3%
- Unequal variances = 11.1%
- Incomplete (co)variation = 82.2%

Nonparametric Analysis



Why nonparametric?



- One might be interested in the comparison of the ranking of realobserved values versus those predicted by models
 - Bull's EPD for efficiency
- More resilient to abnormalities of the data
 - Outliers and influential points

NONPARAMETRIC TESTS

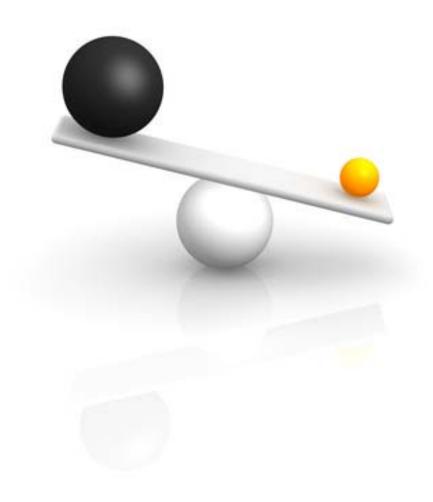
Spearman's r_s is the linear correlation coefficient of the ranks

$$r_{S} = \frac{\sum_{i=1}^{n} (R_{i} - \overline{R})(S_{i} - \overline{S})}{\sqrt{\sum_{i=1}^{n} (R_{i} - \overline{R})^{2}} \sqrt{\sum_{i=1}^{n} (S_{i} - \overline{S})^{2}}}$$

• Kendall's τ measures the ordinal concordance of ½·n· (n-1) data points where a data point cannot be paired with itself

$$\tau = \frac{Concordant - Discordant}{\sqrt{Concordant + Discordant - ExtraY} \times \sqrt{Concordant + Discordant - ExtraX}}$$

Balance Analysis



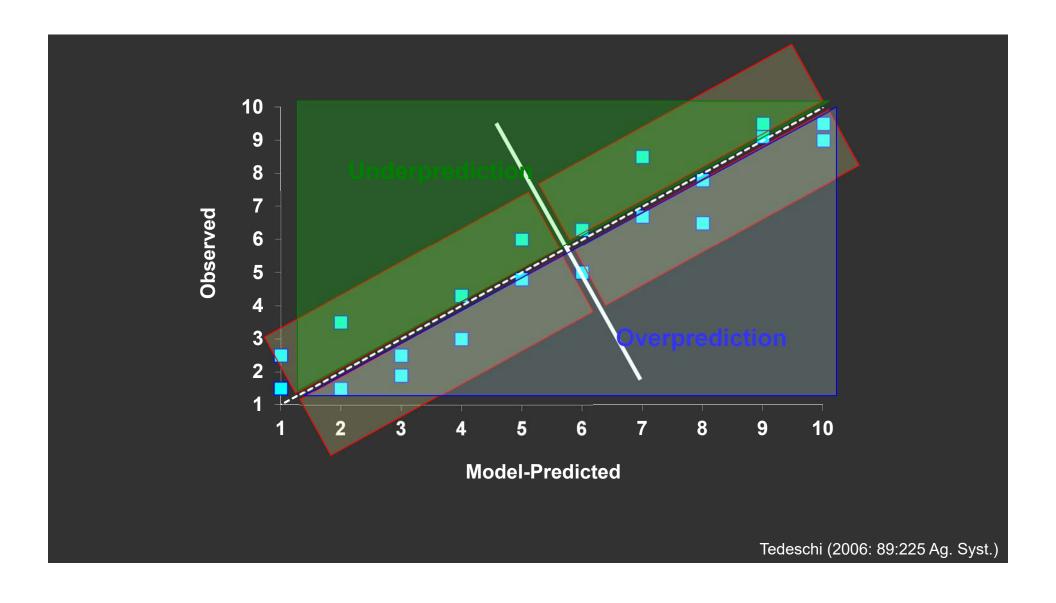
Tedeschi (2006: 89:225 Ag. Syst.)

BALANCE ANALYSIS

 Evaluates the balance of number of data points under- and overpredicted by the model above and below the observed and model-predicted mean, orthogonal or not to the regression line.

Model prediction	Observed or Model-Predicted Mean	
	Below	Above
Overpredicted	n_{II}	n_{12}
Underpredicted	n_{21}	n_{22}

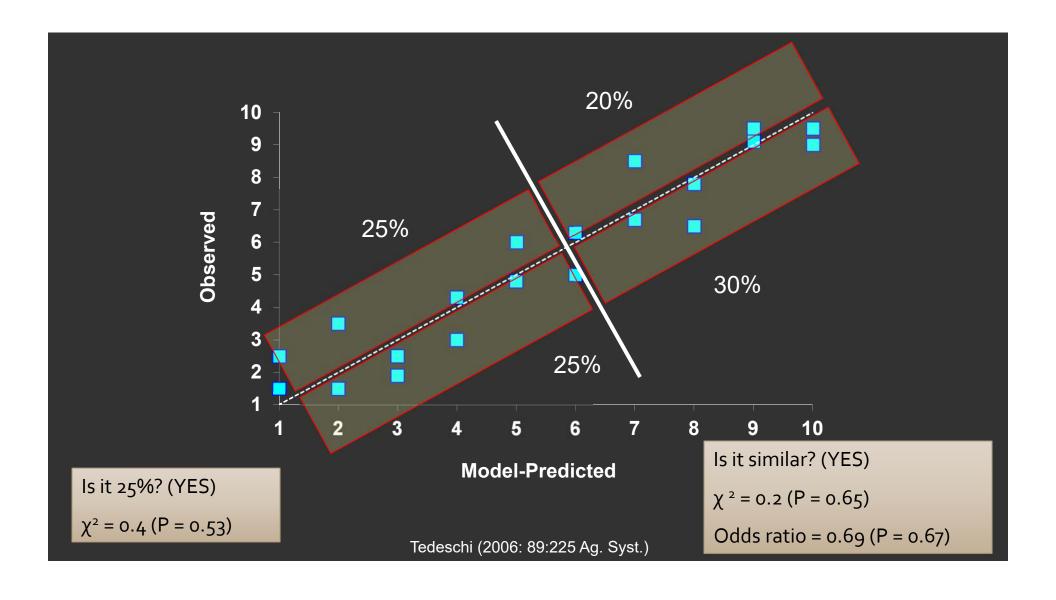
BALANCE ANALYSIS



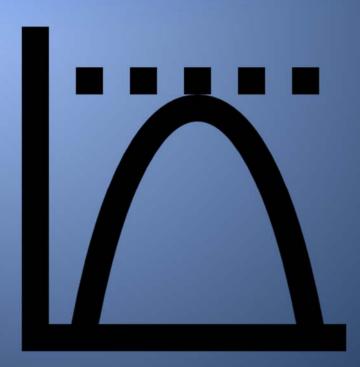
What do we check in the Balance Analysis?

- Is the trend of under- or overprediction similar?
- Is it similar below and above the mean?
- Use χ² analysis to check if the number of points is not different
 - Check if they are 25%
 - Check if the distribution is similar

BALANCED ANALYSIS



Data Distribution



Stochastic models

-- Reynolds and Deaton (1982)

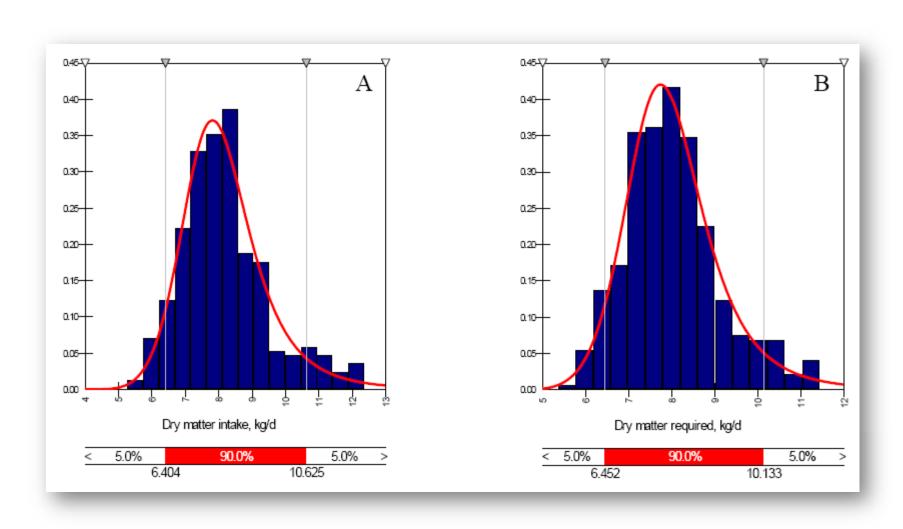
Data Distribution

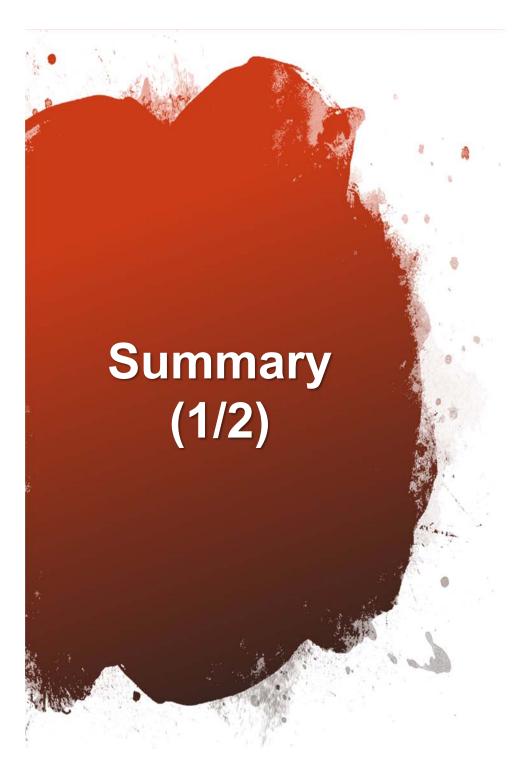
Deterministic models

-- Dent and Blackie (1979)

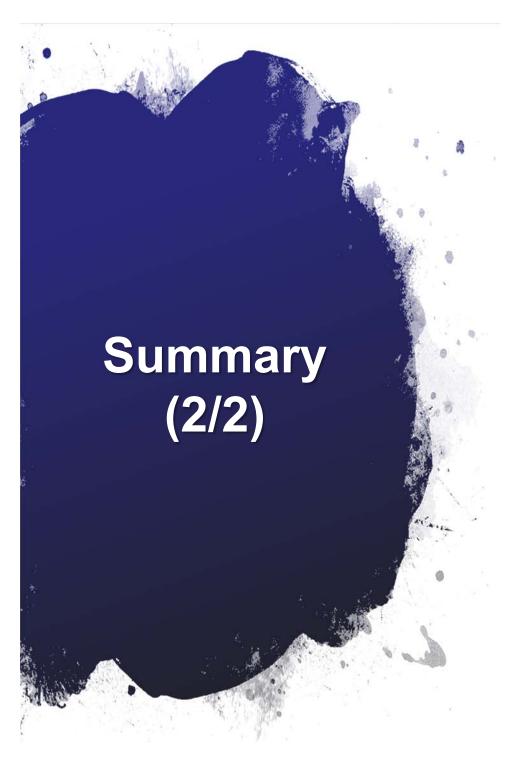
The idea is to check if observed and predicted values come from the same distribution

Data Distribution





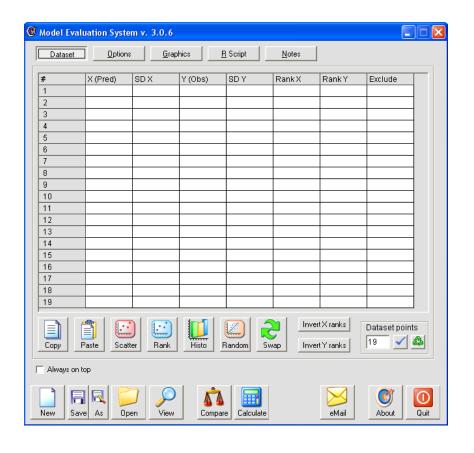
- Acceptance of model wrongness is important to ensure that more reliable and accurate models are developed
- The usefulness of a model depends on the purpose it was developed for

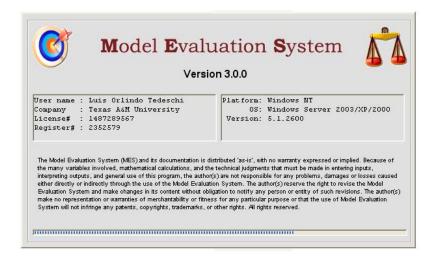


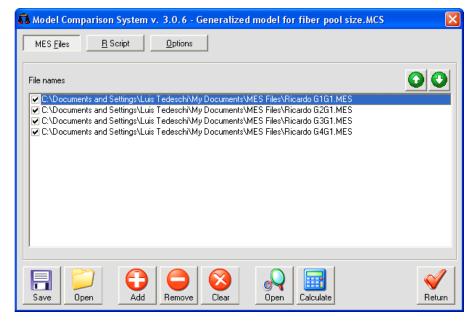
 High accuracy and high precision of a model for a given database implies NOTHING regarding future predictions of the model

 Model evaluation MUST be assessed using several statistical techniques; each technique measures different characteristics of the model

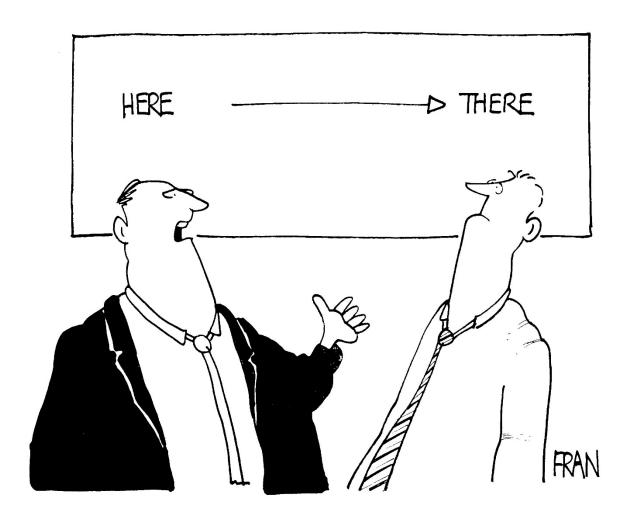
A handy assistant ...







http://nutritionmodels.tamu.edu OR http://www.nutritionmodels.com/mes.html



"It's a simple model... but it works for me..."





KESEAKCH



National Animal Nutrition Program Leveraging Resources, Linking Researchers

